<u>The Six Sigma approach to capability and understanding the 1,5 sigma shift and drift of the process mean</u>

Coming to grips with the 1,5 sigma shift is one of the more interesting challenges facing the Six Sigma Black Belt.

The goal of this article is to explain the reasoning behind this method, made popular by Motorola in the mid 1980's.

The method uses the power of the Standard Normal distribution, which is widely applied to "natural" processes and which is conveniently available in software, algebraic, or tabular form for capability analysis.

Most capability techniques apply the predictive property of this curve to normal data: for example, 99,73% of the area under the curve lies from -3 standard deviations to +3 standard deviations about the mean. If a process has a mean of 10 and a standard deviation of 2, the Standard Normal function predicts that 0,27% of the data points are to be found outside the ±3 standard deviation limits, that is, the area under the curve less than 4 added to the area under the curve greater than 16. These would typically define out of specification limits in process capability analysis.

No adjustments were made to this straight forward use of the Standard Normal function until Motorola led the field by accounting for nature's shifts and drifts. Their engineers moved the calculated mean by 1,5 sigma to adjust for nature's capability.

The 1.5 sigma shift applies a correction to data we get from the production line to better evaluate what our valuable customers really do see. The older capability calculation is well adapted to "short term," "line-side" data, but, because of nature's shift and drift over time, it is overly optimistic about the "long term" capability by which customers judge quality.

This article outlines the treatment of capability metrics with continuous and attribute, or discrete, data and describes the origin and use of dpmo Sigma Quality Level tables.

<u>Continuous data and attribute data are not treated in the same way when calculating Sigma Level</u>

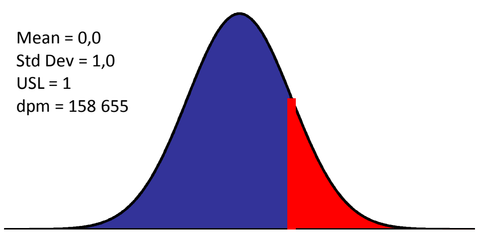
Providing it is random and normally distributed, continuous data uses the Standard Normal distribution, known as the Z function for the determination of capability indices: the number of standard deviations (σ) between the mean and a specification limit are calculated and can be used to predict the probability of having defects. Conversely, attribute data begins with the probability of having defects and works back to estimate capability assuming that the distribution is normal.

<u>The Standard Normal curve is primarily adapted to use with continuous data</u>

The Standard Normal curve has a total area of 1 and a mean of 0. The area under this Gaussian, "bell" curve is positioned symmetrically about 0 and allows calculation of the probability of finding values inside an interval from $-\infty$ (or from the mean, μ) to $\mu \pm (Z \times \sigma)$. The table in Appendix 1 shows the "right sided" Z function distribution giving the total area under the curve from $-\infty$ up to a specification limit corresponding to $\mu \pm (Z \times \sigma)$.

The table gives the area under the Standardized Normal Z function on the basis of a distance from the mean, μ, expressed as ±Z times the standard deviation, σ (Z×σ.) Looking up Z=1; Z=2; Z=3 in the Z (Standard Normal Distribution) table in Appendix 1, we find that, for these values of Z, with σ=1, the areas under the curve are 0,8413; 0,9772; 0,9987, which represent the area under the Standard Normal curve between $-\infty$ and the specification limit at $\pm(Z \times \sigma)$ distance from the mean, μ. These areas under the Standard Normal curve up to a chosen specification limit at $\mu+(Z \times \sigma)$ represent the probability of finding data points providing that the data is normal and randomly distributed. On the left side of a single Upper Specification Limit, the area would be the process yield, and on the right side, would be the proportion of defects. In general, the area under the curve within the specification limits is the predicted yield, and whatever is out of limits is the probability of defects.

Fig 1 Area under the Standard Normal curve up to 1σ



From $-\infty$ to a single upper specification limit of 1×σ (i.e. Z=1) the Z table in gives 0,8413: this Yield of 84,13% means that 84,13% of the probable data points are found under the curve up to 1σ, and that probable defects of 15,87% (or 158.655 defects per million) are found beyond the Upper Specification Limit, USL. (Fig. 1)

The Standard Normal distribution is symmetric about the mean, and has perfectly defined areas under the curve between the various multiples of σ (quantiles) around μ=0.

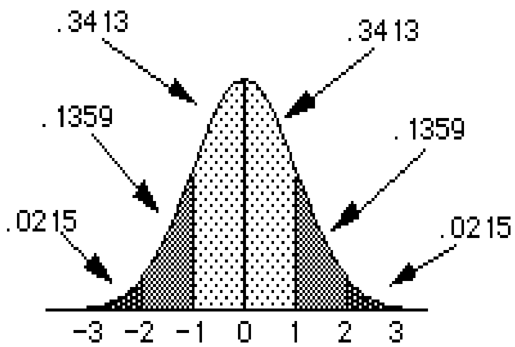


Fig 2

The Z distribution table needs to be read depending on which half of the symmetrical curve in Fig 2 has been written into it. In a two-sided calculation, defects would be calculated and added together both from the left side inferior to the LSL and from the right side beyond the USL. Two sided values need to be interpreted from the table in the following way:-

$$\pm 1\sigma = 2\times 0,3413 = 68,26\%(Defects\,31,74\%;317.400dpm)$$
$$\pm 2\sigma = 2\times 0,4772 = 95,44\%(Defects\,4,56\%;45.600dpm)$$
$$\pm 3\sigma = 2\times 0,4987 = 99,74\%(Defects\,0,26\%;2.600dpm)$$

If μ is half way between the Lower Specification Limit and the Upper Specification Limit and exactly half of the expected defects are situated to the left of the LSL and the other half to the right of USL, this is the special case where the process distribution is perfectly centered. In this case, as we shall see below, C_p, the Capability "potential" index and the maximum possible value for C_{pk}, the Capability "centering" index, have the same value.

The Z table in Appendix 1 does not give values beyond 3σ, but, a full table or software would give a two sided value at ±6σ of:

$$\pm 6\sigma = 99,9999998\%(Defects\,0,0000002\%;0,002dpm)$$

<u>The Standard Normal function leads to C_p and C_{pk} capability metrics</u>

In a perfectly centered "6σ" process, μ is defined to be at a distance of 6xσ (i.e. Z=6) from both the USL and the LSL. In this case, the C_p (potential capability) value is 2, and is calculated as:

$$C_p = \frac{|USL - LSL|}{6\sigma} = \frac{12\sigma}{6\sigma} = 2$$

The C_{pk} value is also 2 for a centered process and is calculated as :

$$C_{pk} = MIN\,\frac{|USL-\mu|}{3\sigma};\frac{|\mu - LSL|}{3\sigma} = \frac{6\sigma}{3\sigma} = 2$$

Here, specification limits representing the "Voice of Customer" are defined as being 12σ apart. The 6σ denominator in C_p representing the "Voice of Process" is calculated from the process data and chosen by convention to be +3σ-(-3σ)=6σ, giving, as we saw above, 99,74% of all process data, (providing that the data is normally distributed.)

The C_{pk} calculation uses the half value of [USL-LSL] when the process is centered, otherwise it is derived from the distance between μ and the closest specification limit.

<u>Long term capability integrates a 1,5σ shift and drift of μ</u>

Evaluating capability of a process over a short period of time does not always account for "long term" shift and drift of its mean. Research shows that all processes have natural shift and drift that "short term" metrics do not show up. Motorola quotes the work of three researchers, Bender (1975,) Evans (1975) and Gilson (1951,) who agree among themselves that the "long term" shift and drift of the mean in natural processes is 1,5σ around the "nominal" μ.

If the mean drifts 1,5σ towards one of the specification limits, then C_p (the potential) does not change, but the centering index does change, so C_{pk} becomes:

$$C_{pk} = MIN\,\frac{|-7,5\sigma|}{3\sigma};\frac{|4,5\sigma|}{3\sigma} = \frac{4,5\sigma}{3\sigma} = 1,5$$

Motorola therefore defines '6σ' quality to integrate shifts and drifts of the mean, and defines the double condition for a '6σ' process to be C_p=2 <u>and</u> $C_{pk}\geq$1,5.

The two graphs in Fig 3 illustrate the impact of the drift of the mean on C_{pk} where C_p does not change. They show that the effect of the 1,5σ drift of the mean is to place μ at a distance of 4,5σ from the closest specification limit in the case of a C_p=2 process, and that, in this case, C_{pk} is reduced from 2 to 1,5. The defect level increases from 0,002 dpm at a Sigma Level of 6 to 3,4 dpm because the Z function gives 3,4 dpm, or 99,99966% yield, with a Sigma Level at 4,5σ from the mean. Motorola have established a minimum

C_{pk} of 1,5 as allowable for a 6σ (Cp=2) process because of this unavoidable shift and drift of the mean: a '6σ' process will have up to 3,4 dpm to account for long term variation, and not 0,002 dpm, which is a short term value.

The Poisson distribution can be used to approximate First Pass Yield with attribute data

If a batch of 1.000 products contains 4 defects. The overall ratio of errors per product is called "defects per unit" (dpu,) and has a value, in this case, of dpu=4/1.000=0,004

Providing that the errors are randomly distributed, the Poisson distribution is used to predict how many of the products have 0;1;2;3, etc. errors. The formula calculates a yield (Y) on the basis of dpu and , r, the chosen number of errors per product being evaluated.

$$Y = \frac{dpu^r e^{-dpu}}{r!}$$

For example, if pens coming off the end of a production line show a reject rate of 0,004 dpu, then, for r=0, the Poisson function shows per 1.000.000 pens produced, 996.008 defect free pens; for r=1, it shows 3.984 pens with 1 defect, and for r=2, it shows 8 pens with 2 defects.

In the special case where r=0, the yield representing the proportion of defect free units passing the first time without any defects whatsoever through the entire process is called the First pass Yield (FPY). Putting r=0 in the Poisson expression gives two useful expressions for estimating FPY and dpu from each other:

$$FPY = e^{-dpu} \qquad dpu = -\ln FPY$$

Because FPY is the product of the individual yields at each process step, we derive:

$$FPY = Y_A \times Y_B \times Y_C \times .....Y_n$$

and for n individual process steps, we can estimate the average yield of each process step as:

$$Y_i = \sqrt[n]{FPY}$$

First Pass Yield allows the calculation of Six Sigma process levels

The formulae shown above are typically used to compare and target capability levels. For example, defect data was collected across a batch of 1.500 electronic devices made in a 15 step process, and after collection of the data, 325 defects were identified.

From this information, we calculate the overall process dpu:

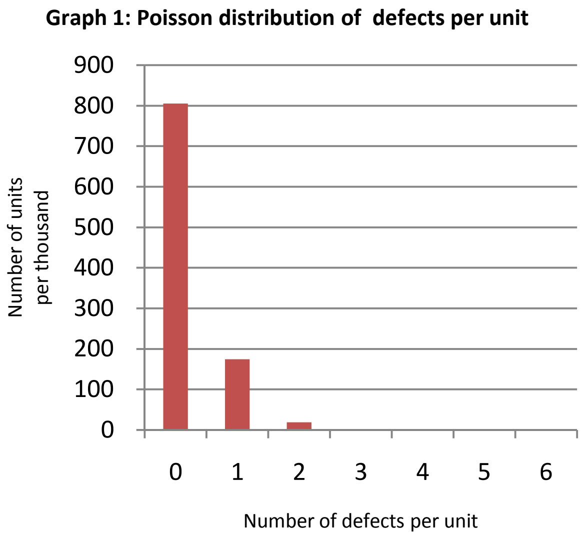
$$dpu = \frac{325}{1500} = 0,2167$$

From the process dpu, we derive the overall First Pass Yield:

$$FPY = e^{-dpu} = e^{-0,2167} = 0,8052$$

This equates to a rate of 805.200 conforming first pass units per million, corresponding to the r=0 defects in the Poisson distribution formula, and illustrated in the first column on Graph 1 below: the other values are derived from the Poisson distribution formula for different values of r, at r=1, 174.500 non conforming first pass units with 1 defect; r=2, 18.900 non conforming with 2 defects; r=3, 1.400, with 3 defects.

From Appendix 1, the sigma level for this FPY is 0,86. Adjusting for shifts and drifts gives a long term FPY Sigma Level of 0,86+1,5=2,36, and a C_{pk} of 2,36σ/3σ=0,79. This is the global capability of the whole process to produce products right the first time.



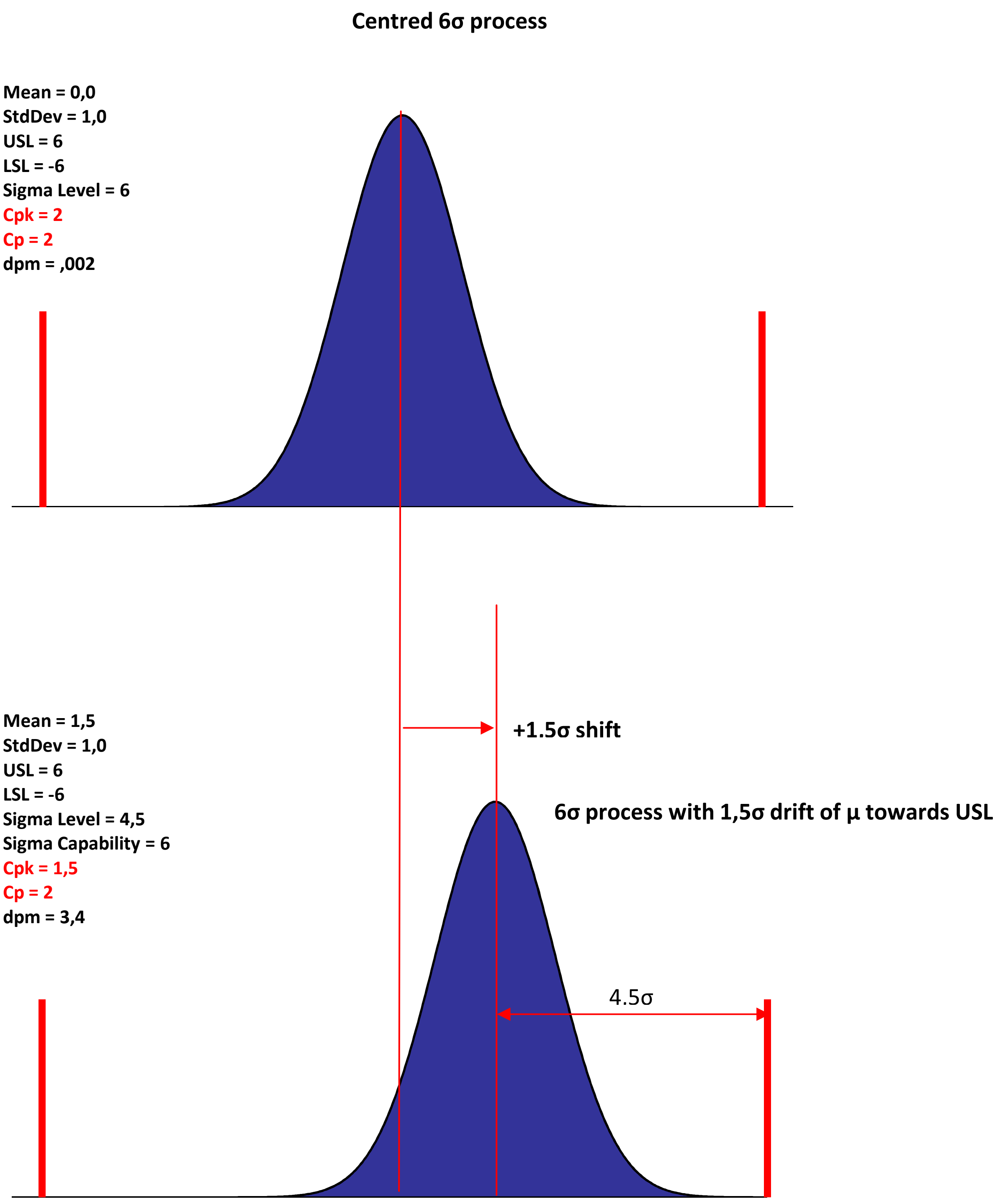

Centred 6σ process
Mean = 0,0
StdDev = 1,0
USL = 6
LSL = -6
Sigma Level = 6
Cpk = 2
Cp = 2
dpm = ,002
Mean = 1,5
StdDev = 1,0
USL = 6
LSL = -6
Sigma Level = 4,5
Sigma Capability = 6
Cpk = 1,5
Cp = 2
dpm = 3,4
+1.5σ shift
6σ process with 1,5σ drift of μ towards USL
4.5σ

Fig3

In this 15 step process, the expected average yield per process step would be:

$$Y_i = \sqrt[n]{FPY} = \sqrt[15]{e^{-0,2167}} = 0,9857$$

This corresponds to a Sigma Level of 2,19, to which we add 1,5σ to include shifts and drifts: at the average individual process level this gives 3,69σ, and, dividing by 3σ, gives an average individual process level C_{pk} of 1,23. A '6σ' process step would be required to have a C_{pk} of at least 1,5.

We can compare the overall process metric with the long term process capability we would achieve from 15 process steps of 6σ quality. If we use the best known 1,5σ shifted, one sided Z function goal of 3,4 dpm, then FPY is :

$$FPY = Y_A \times Y_B \times Y_C \times .....Y_n$$
$$FPY = 0,9999966^{15} = 0,999949$$

This gives the long term process Sigma Level for 15 '6σ' process steps as 3,89σ, which, dividing by 3σ, is a C_{pk} of 1,3. We found that the overall process C_{pk} is 0,79, which compares unfavorably with this, and confirms, as we saw from the individual process steps, that this is not a '6σ' process.

It means that the current yield must be improved by using 6σ tools, knowing that only a part of the difference may correspond here to pure machine performance, the rest being dependent on material quality, shelf life, procedures, etc.

dpmo and Sigma Level tables are used to calculate the capability of attribute data

The 0,002 dpm value does not correspond to the 99,99966% (defect rate of 3,4 dpmo) commonly stated for a 6σ process. The Sigma Level of 0,002 dpm is a two sided value calculated directly from the Z function, whereas the 3,4dpmo commonly associated with '6σ' capability is:-

- ✓ a one sided value
- ✓ that includes a 1,5σ shift of μ, the mean.

Capability calculations for attribute data start with the calculation of dpmo, defects per million opportunities, and work back to calculate a Sigma Quality Level.

From the example above, if a batch of 1.500 electronic devices had 325 defects, and any unit is shown to have 5 types of defect on any of the 35 parts, then, because there are 1.500x35x5 opportunities to make defects, the dpmo would be calculated as:

$$dpmo = \frac{325}{1.500 \times 35 \times 5} \times 1.000.000 = 1.238$$

Looking up how many σ's in the Z table correspond to 1.238 dpm, we find a Sigma Level of 3σ, to which we add 1,5σ to account for shifts and drifts. Dividing the total 4,5σ by 3σ gives a C_{pk} of 1,5, which is far from the C_{pk}=0,79 given by using the dpu calculation. Clearly this is so because we have inflated the denominator of the expression.

As an 'opportunity' is a binary event, we only use the dpmo term with attribute data. In practice, attribute data cannot generally be ascribed to one side or to the other of a Z distribution, and Motorola has taken as convention that the dpmo used to calculate attribute Sigma Quality Level would be one sided and to the right of the Z distribution.

To calculate capability indices, firstly we ask whether the defects are one sided or two sided: that is to say, do we consider that there are 1.238dpmo above the Upper Specification Limit, or do we consider there are 619dpmo below the LSL <u>and</u> 619dpmo above the USL? In this case, where an error is a binary, attribute, event, 1.238dpmo is assumed, in accordance with Motorola's guidance, to be to the right of the USL.

In essence, calibrating the <u>proportion</u> of defects (dpmo) to the right side tail area of the Z function allows attribute data to be evaluated with the <u>continuous</u> Standard Normal distribution.

Commonly available yield tables confuse continuous two sided or one sided Z function dpm values with one sided dpmo values shifted by 1,5σ. To limit the confusion, some practitioners speak of "Sigma Level" for continuous data and of "Sigma Quality Level" for shifted attribute data. Examples are shown in Table 1 and Graph 2 below.

Continuous data					Attribute data	
Sigma Level	Cp for a centred process	dpm: 2 sided spec limits: mean not shifted	Cpk	dpm: 1 sided spec limit: mean not shifted	Sigma Quality Level	dpmo: 1 sided spec limit: mean shifted by 1,5σ
1	0,33	317 310	0,33	158 655	1	691 462
2	0,67	45 500	0,67	22 750	2	308 538
3	1,00	2 700	1,00	1 350	3	66 807
4	1,33	63,4	1,33	31,7	4	6 210
5	1,67	0,574	1,67	0,287	5	233
6	2,00	0,002	2,00	0,001	6	3,4
7	2,33	0,0000026	2,33	0,0000013	7	0,019

Table 1

Table 1 uses the Z function and can lead to confusion because the 1,5 sigma shift introduced into the attribute Sigma Quality Level on the right hand side is not explicitly included in the continuous Sigma-Level data on the left hand side. The dpmo values in the shifted attribute data on the right side are nothing more than, as we can see in Table 2 , those we would get from the unshifted C_{pk} values if we reduce the Sigma Level by 1,5. This verifiable from Appendix 1, but the dpm values have been derived with more precision using software.

Continuous data			Attribute data	
Sigma Level	**Cpk**	**dpm: 1 sided spec limit: mean not shifted**	**Sigma Quality Level**	**dpmo: 1 sided spec limit: mean shifted by 1,5σ**
-0,5	-0,17	691 462	**1**	691 462
0,5	0,17	308 538	**2**	308 538
1,5	0,50	66 807	**3**	66 807
2,5	0,83	6 210	**4**	6 210
3,5	1,17	233	**5**	233
4,5	1,50	3,4	**6**	3,4
5,5	1,83	0,019	**7**	0,019

Table 2

A commonly seen graphical representation of these tables is shown in Fig 4.

Why did Motorola propose this approach of comparing a two sided Z value of 0,002 dpm for a centered 6σ process with 3,4 dpmo for a one sided Z value adjusted for a 1,5σ shift of μ?

The broad reason is that this forces the addition of a 1,5σ change in the population "long term" mean when necessary. 0,002 dpm is intended to represent the "short term" steady state level seen with continuous data, and 3,4 dpmo represents the "long term" dynamic, real world state of affairs written in already to the attribute curve. If we used batched components in our process, over a very large number of batches we would expect a movement of the mean of 1,5σ: we compensate for this by taking short term data and adding the drift to the mean to calculate the Sigma Quality Level. A defect proportion of up to 3,4 dpmo in a C_p=2 process is still considered to be a "6σ" process, the shift accounting for long term variation of noise factors, such as temperature, humidity, tool wear, variation in raw materials or in parts, etc.

The Motorola approach helps us to understand and to better use capability metrics: its merit is twofold:

1. in adapting the use of the Z function to evaluate the capability of attribute data, and
2. in reminding us that process shift and drift require us to adjust short term data before attempting to compare long term Sigma Quality.

There is some difficulty in reconciling dpmo and dpu calculations of capability, and it is better to go either with one system or the other. Establishing how many ways a product or service can fail gives scope to a lot of uncertainty in the denominator of the dpmo expression. For this reason, its use is sometimes confined to tracking improvements or to comparing processes of different complexity. For example, the dpu of producing a light bulb or of a jet engine may be the same, but the different complexity of these two products imply very different dpmo's which we are forced to take into account if we need to understand and compare differences in C_{pk} between business units, for example.

Using dpu to calculate C_{pk} does not allow comparison of the complexity of processes the way dpmo does, and gives Sigma Levels that would not account here for the added complexity of multiple categories of defect across multiple components. However, dpu is easier to use and to understand, it answers the major questions, it uses the Poisson function understandably, and avoids endless discussions in the six sigma team about the ways a process can go wrong.

The best advice is to choose one or the other according to the need, and to stay consistent. Mixing dpmo and dpu calculations can confuse.

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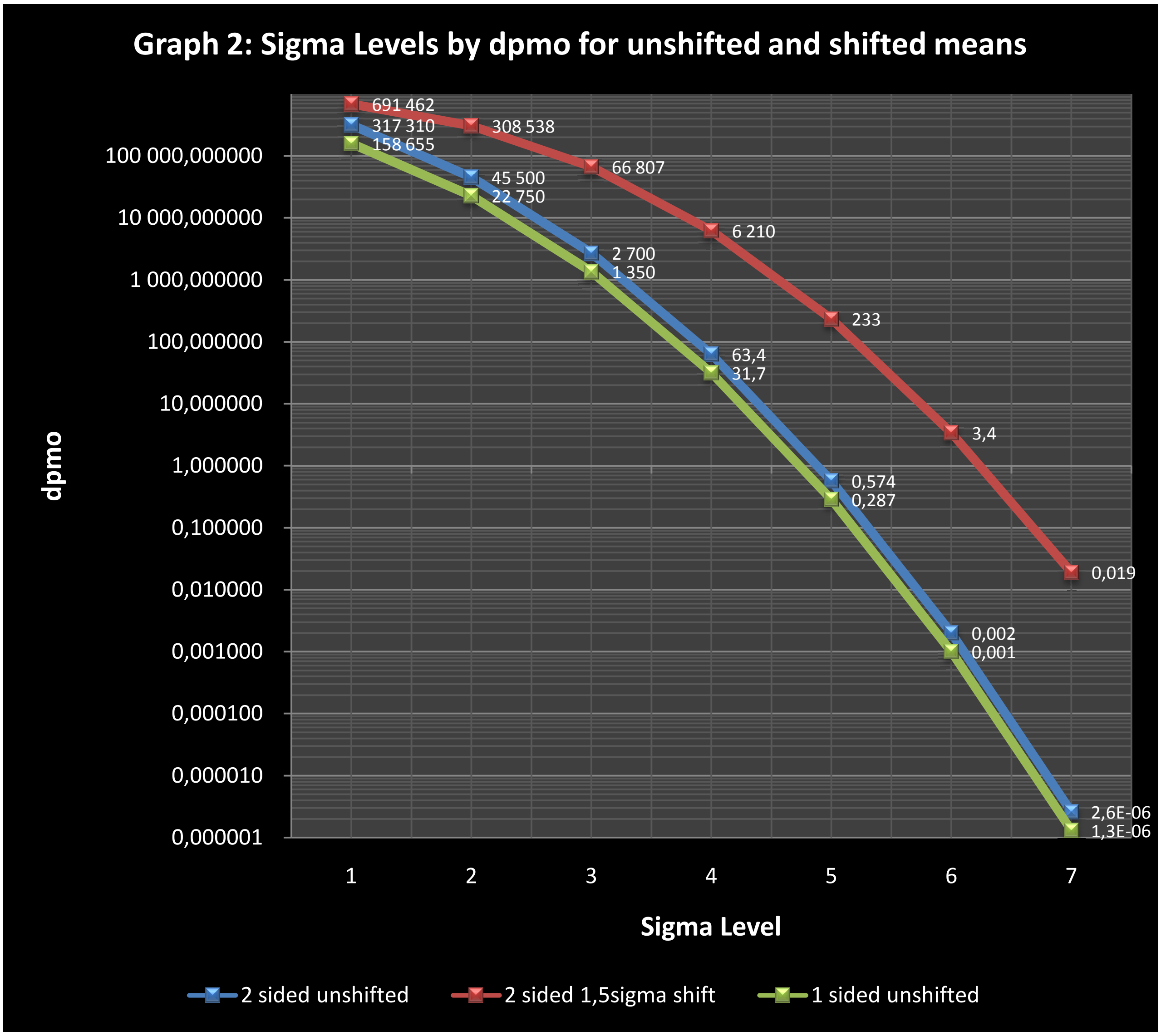
Graph 2: Sigma Levels by dpmo for unshifted and shifted means
dpmo
100 000,000000
10 000,000000
1 000,000000
100,000000
10,000000
1,000000
0,100000
0,010000
0,001000
0,000100
0,000010
0,000001
691 462
317 310
158 655
308 538
45 500
22 750
66 807
2 700
1 350
6 210
63,4
31,7
233
0,574
0,287
3,4
0,002
0,001
0,019
2,6E-06
1,3E-06
1
2
3
4
5
6
7
Sigma Level
2 sided unshifted
2 sided 1,5sigma shift
1 sided unshifted

Appendix 1

The Standard Normal (*Z*) Table

1. Values in the table represent areas under the curve to the left of *Z* quantiles along the margins. Examples: $z_{.5000}$ = 0.00; $z_{.9750}$ = +1.96; $z_{.0250}$ = -1.96

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990